



# Let's Talk Math

Engaging Students as Mathematical Thinkers to Deepen  
Problem-Solving Skills through Mathematical Discourse

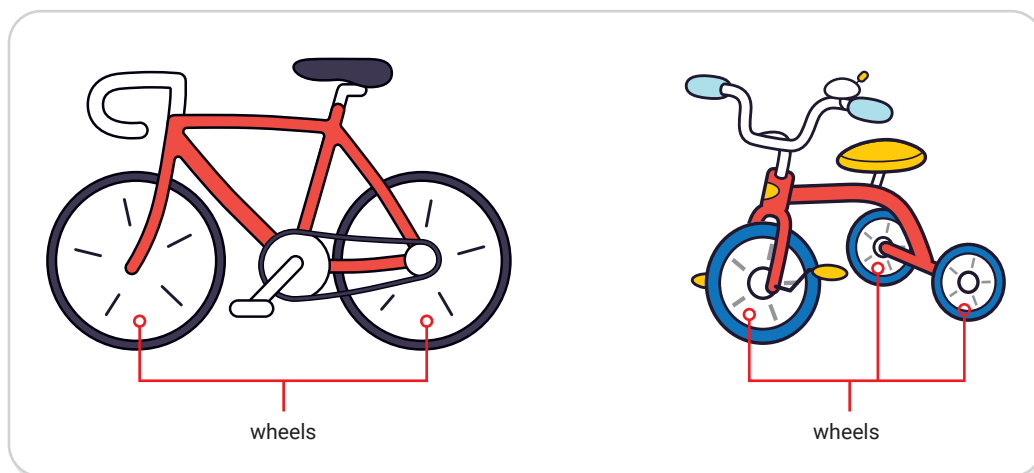
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# Introduction

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A mathematics coach at an elementary school met with a team of third-grade teachers from the same school, all of whom follow the same curriculum. The purpose of the meeting was to select a high cognitive demand task and discuss implementation strategies. Collectively, the teachers decided to assign the following task:



Bill's shop sells bicycles and tricycles. Bicycles have 2 wheels. Tricycles have 3 wheels. Bill counted 18 total wheels in his shop. How many bicycles and tricycles could Bill have in his shop? Explain your thinking.

The teachers intentionally chose a task that was non-routine and required critical thinking and perseverance to be able to solve.

A week later, the Grade 3 team met again to discuss their student work on the Bicycle-Tricycle task. While Teachers 1 and 2 brought work that demonstrated a range of approaches, Teacher 3 determined that she didn't think her students could do this task successfully, so she adjusted it to make the task more "accessible." She told her students that there were 5 total wheels instead of 18, justifying that 5 was a friendlier number. While Teacher 3 had good intentions to support her students, by changing the task she ultimately lowered the **cognitive demand** and reduced (or possibly eliminated) the opportunity for her students to **productively struggle**. Her students were not given the same opportunities as the other third graders to make sense of the task, apply their mathematical skills to solve the task, and then justify their conclusion. It also became apparent through this grade-level discussion that, overall, students struggled to start the process of problem-solving independently and without teacher support.

Teaching problem-solving is complex and challenging. It's not called *problem solving* because it's easy!

Realistically, educators will never be able to fully prepare students for all the types of problems they will encounter. Consequently, educators need to offer students strategies to engage in mathematical thinking that can be applied to a vast range of contexts.

Additionally, they must provide structure for students to know how to engage in effective mathematical discourse with peers, and to then work independently to attack a problem so they can discuss and compare their approaches.

All mathematics educators want students to become confident and independent mathematical thinkers. So, how can educators accomplish this objective? *Let's Talk Math* sets out to answer that question and presents a comprehensive approach to engage students as mathematical thinkers.

→ **cognitive demand:** the required level of thinking initiated through making sense of a task

→ **productive struggle:** the act of persevering by approaching a task in which the solution isn't immediately obvious and attempting to work out solutions

## About the Authors

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As former elementary educators, Kit Norris and Dr. Hilary Kreisberg understand and empathize with teachers who are challenged to be generalists and are expected to teach multiple content areas while also maintaining a strong content knowledge in each. It is not an easy feat for teachers to lead a student-centered classroom if they lack confidence themselves in a particular subject or are not quite sure what the standards really require. Part of Norris and Kreisberg's intent for this supplemental resource is to provide teachers with predictable and structured routines that are easy to implement and can support any mathematics curriculum. They specifically felt drawn to develop routines that integrate mathematics and literacy since these two content areas overlap in many ways. Students should be able to speak, listen, read, and write mathematically. *Let's Talk Math* supports students as they learn how to think mathematically and helps raise their self-confidence as they become successful problem solvers. In addition, students need to be presented with challenging, rich tasks that offer high levels of cognitive demand and rigor. The tasks in *Let's Talk Math* are designed to engage students in making connections between mathematical ideas and facing challenges that require perseverance. This resource requires students to monitor their own processes of thinking and analyze their own work and the work of their peers.

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**Hilary Kreisberg, Ed.D.**, is the director of the Center for Mathematics Achievement at Lesley University. She is the author of several mathematics education books, a frequent national, regional, and local speaker and has won millions of dollars in federal and private funding for mathematics education research. Before becoming a university professor and professional development provider, Kreisberg was a K–5 math coach and an elementary educator. She has a Doctor of Education degree in Educational Leadership and Curriculum Development, a Master of Arts degree in Teaching and Special Education, and a Bachelor of Arts degree in Mathematics. Kreisberg is also a certified U.S. Math Recovery® Intervention Specialist with specialization in number learning, and holds an endorsement for Sheltered English Immersion (SEI) instruction.

## The Logic Model

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The Logic Model in Figure 1 demonstrates how *Let's Talk Math* is designed to develop lifelong mathematical problem solvers. Evidence of this is suggested through its resources and activities, which are linked to positive outcomes for students. The goal of this table is to help the reader visualize how implementing *Let's Talk Math* can support and contribute to achieving school and district goals.



Figure 1—Logic Model Guiding Principles

<b>Problem Statement: There is a need for evidence-based pedagogical strategies to aid mathematics educators in effectively teaching mathematical problem-solving.</b>					
<b>Outcome/Goal: To help students engage as productive mathematical thinkers</b>					
<b>Theory of Action</b>					
Educators implement evidence-based mathematical strategies.	K–5 students engage in and utilize mathematical strategies.	K–5 students will have increased skills and comfort with mathematical practices and processes.	K–5 students will have increased achievement in mathematical problem-solving.	K–5 students will be prepared for secondary and post-secondary education success.	Students become lifelong mathematical problem-solvers.
<b>Logic Model</b>					
<b>Assumptions</b>	<b>Resources/Inputs</b>	<b>Activities</b>	<b>Outputs/Metrics</b>	<b>Outcomes</b>	<b>Impact</b>
<ul style="list-style-type: none"> <li>• Students can learn multiple new practices and processes of mathematical thinking and analyze them in math tasks.</li> <li>• Mathematical problem-solving is important for school success.</li> <li>• School districts are interested and prepared to incorporate new and different teaching strategies.</li> </ul>	<ul style="list-style-type: none"> <li>• Based on extensive mathematics research</li> <li>• Protocols to structure students' mathematical discourse</li> <li>• 120 student tasks</li> <li>• 12 Instructional videos</li> <li>• 5 Professional Development videos</li> <li>• Teacher's Guide</li> <li>• Student Task Book</li> <li>• Digital Resources</li> </ul>	<ul style="list-style-type: none"> <li>• Teach students specific protocols to support their mathematical discourse and problem-solving (Understand and Plan; Share and Discuss; Reflect and Write).</li> <li>• Teach students to make sense of problems and persevere in solving them.</li> <li>• Teach students to reason abstractly and quantitatively.</li> <li>• Teach students to construct viable arguments and critique the reasoning of others.</li> <li>• Teach students to mathematize a situation.</li> <li>• Teach students to use appropriate tools strategically.</li> <li>• Teach students to attend to precision.</li> <li>• Teach students to look for and make use of structure.</li> <li>• Teach students to look for and express regularity in repeated reasoning.</li> </ul>	<ul style="list-style-type: none"> <li>• Expressive mathematical problem-solving growth</li> <li>• Meet or exceed expectations of mathematics standards</li> <li>• Completion of tasks</li> <li>• Robust mathematical discourse</li> </ul>	<ul style="list-style-type: none"> <li>• Knowledge of mathematical practices and processes</li> <li>• Deepened overall mathematics awareness</li> <li>• Ability to engage in mathematical discourse and mathematical problem-solving</li> <li>• Increased achievement in mathematics standards</li> <li>• Engagement in mathematics</li> </ul>	<ul style="list-style-type: none"> <li>• Increased mathematical thinking, discourse, and problem-solving among students</li> <li>• Prepared for secondary education success</li> </ul>

# Guiding Principles

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Two guiding principles informed the development of this resource. *Let's Talk Math* is built on the premise that:

1. High-quality mathematics instruction prioritizes implementing meaningful, high cognitive demand tasks, engaging students in using habits of mathematical thinkers, and supporting all learners.
2. Communicating mathematically encompasses speaking, listening, reading, and writing effectively.

These guiding principles are the foundation of *Let's Talk Math* and are embedded in each and every component of the product.

**Guiding Principle #1:** High-quality mathematics instruction prioritizes implementing meaningful, high cognitive demand tasks, engaging students in using the habits of mathematical thinkers, and supporting all learners.

*Let's Talk Math* kits include 120 nonroutine, high cognitive demand mathematics tasks per grade level that are designed to focus on a specific standard of mathematical practice/process. The tasks use grade-level mathematics content as the vehicle for students to think mathematically. *Let's Talk Math* kits also include teacher's guides with specific instructional guidelines, solutions, possible misconception(s), scaffolding suggestions, extension questions, and language supports.

## Implementing Meaningful, High Cognitive Demand Tasks

One of the principle features of *Let's Talk Math* is its high-quality, rich tasks. Meaningful, high cognitive demand tasks are often sprinkled through traditional core curriculum, omitted altogether, or provided as extension opportunities, thereby making them inaccessible to all students. Since evidence shows that problem-solving using academically challenging tasks with a focus on reasoning offers the greatest learning opportunities for students, it is critical that all students have access to these types of tasks (Smith & Stein 1998).

**nonroutine math task:** a task that requires creative- and critical-thinking skills to solve and enables students to demonstrate their thinking



Take a look at The Closet Floor task from the Grade 3 *Let's Talk Math* kit:

Think Using Quantities



## Closet Floor

Here is a board that is going to be used on the floor in Patrick's new closet.



Patrick wants to cut this board into 4 pieces. He wants them to be equal in size. How can Patrick cut the board? How much of the whole board will each piece represent?

Find a second way Patrick can cut the board to make 4 equal-sized pieces. How much of the whole board will each piece represent?

Extend your thinking!

Now, Patrick wants to partition the same-sized board into 8 equal pieces. How much of the whole board will each piece represent?

Imagine Patrick has a board that measures 9 ft. by 3 ft. He wants to partition it into 3 equal pieces. How much of the whole board will each piece represent?

This task is strategically designed to help students recognize that there are often quantities in a problem that may or may not be important, and to connect the content of geometry with fractions. At its core, the task is simply asking students to identify fourths and to notice that one of four equal pieces is one-fourth of the whole. However, the task is written nonroutinely; it doesn't directly ask for area or perimeter using the labeled units. It requires applying multiple content skills, it has multiple solution paths, and there are multiple solutions ( $\frac{1}{4}$  as one, but also 4 sq. feet as another)—all of which can be challenging for third graders. Because students are so familiar with routine tasks, when presented with this task during pilot testing, many third grade students began finding the area, or the perimeter, or sometimes both, without understanding how they might use that information to answer the question.

Students can only learn how to problem-solve when given tasks that engage them in productive struggle.

Returning to the Grade 3 team meeting described in the Introduction, there are elements of high cognitive demand inherent to the Bicycle-Tricycle task (e.g., multiple solutions; various solution paths; nonalgorithmic thinking and processing of task constraints). As students engage in problem-solving, they will show ranges of high cognitive demand thinking. For example, students who attempt this task may reason that the shop could have 9 bicycles and no tricycles (18 wheels) or 6 bicycles and 2 tricycles (18 wheels), but not 8 bicycles and 1 tricycle (19 wheels). By tackling this task, students are forced to engage with a complex situation where the answer isn't immediately obvious. When Teacher 3 adjusted the task to say that the shop had 5 wheels in total, the cognitive demand of the task was significantly reduced. Multiple solutions had been simplified to one correct answer, the answer was apparent without much effort, and there were no answer restrictions (which is what produced the challenge originally).

### Engaging Students in Using the Habits of Mathematical Thinkers

Mathematical thinkers are problem solvers, not problem performers. *Let's Talk Math* is based on the premise that students:

- work creatively by sense-making and persevering through tasks;
- think critically and reason about a task before calculating and solving;
- collaborate to hear others' ideas to inform decisions;
- communicate by justifying processes and verifying solutions.



Teachers must prepare students for the future. This means training students to be mathematical thinkers by utilizing skills that are important to their future success. *Let's Talk Math* was crafted using the following as theoretical frameworks:

- "Habits of Mind: An Organizing Principle for Mathematics Curricula" (Cuoco, Goldenberg, and Mark 1996)
- NCTM 2000 Process Standards
- National Research Council's 2001 *Adding It Up Proficiency Standards*
- 2010 College and Career Readiness Standards for Mathematical Practice

Though the notion of the habits of mathematical thinkers has been around for some time, it wasn't until the College and Career Readiness Standards for Mathematical Practice—as well as other mathematical process standards, such as those included in the Texas Essential Knowledge and Skills (TEKS) for Mathematics—were implemented that educators began to understand the impact of research meeting practice. Today, teachers all over the country post standards for mathematical practice and processes on their classroom walls and are often looking for ways to build opportunities for students to engage in these practices and processes.

Figure 2 shows how the mathematical practice and process standards are used in *Let's Talk Math*.

Figure 2—Mathematical Practices/Processes

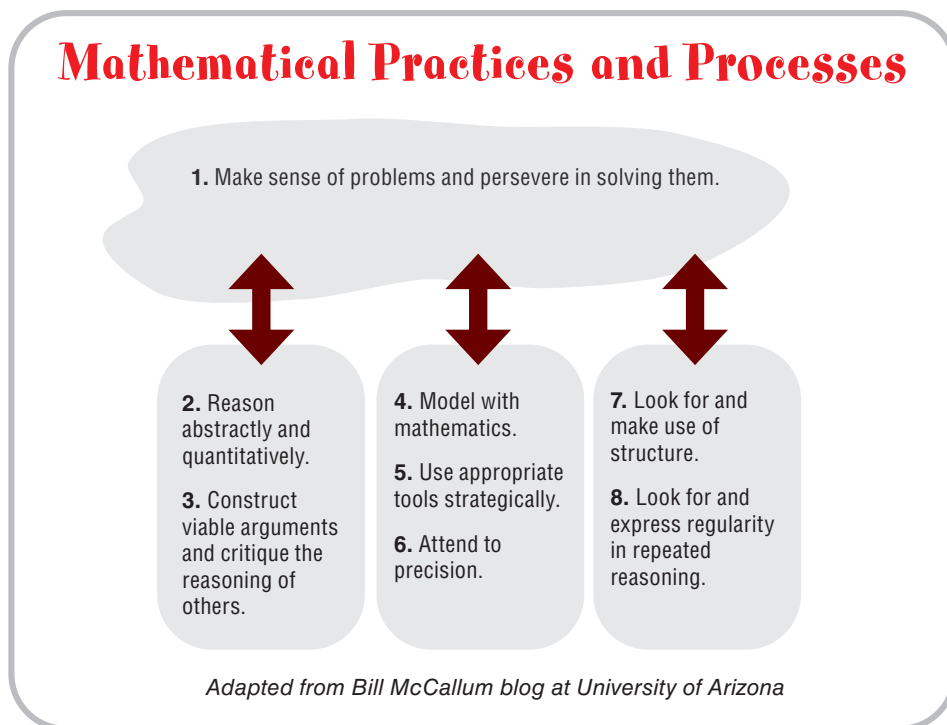


Figure 2—Mathematical Practices/Processes used with permission. From *Engage in the Mathematical Practices: Strategies to Build Numeracy and Literacy with K–5 Learners* by Kit Norris and Sarah Schuhl. Copyright 2016 by Solution Tree Press, 555 North Morton Street, Bloomington, IN 47404, 800.733.6786, SolutionTree.com. All rights reserved.

As shown in Figure 2, sensemaking and persevering, along with attending to precision, are built into every mathematical task in *Let's Talk Math* and are supported by the three routines discussed on page 16 of this paper. Students always begin the problem-solving process by making sense of the question and context that has been presented in the task. Similarly, students attend to precision as they use the appropriate mathematical vocabulary and consider the possible range of values that would make sense in the context of the problem. Additionally, students must persevere through every task. The teacher pages for each task include scaffolding questions and ideas to support students who may be stuck, as well as activities and questions to extend students' thinking.

While every task engages students in making sense of problems, persevering, and attending to precision, the following mathematical practices/processes are specifically highlighted in the tasks and routines, as these are often more challenging to teach:

Let's Talk Math	Mathematical Practices/Processes
Think Using Quantities	Reason abstractly and quantitatively.
Construct and Critique Arguments	Construct viable arguments and critique the reasoning of others.
Mathematize the Situation	Model with mathematics.
Use Tools Strategically	Use appropriate tools strategically.
Analyze the Structure	Look for and make use of structure.
Generalize Your Thinking	Look for and express regularity in repeated reasoning.

Although most curricular resources focus on the mathematics content, *Let's Talk Math* centers on the practices/processes of mathematical thinking, using the content as a tool for exploration. As Sammons (2018, 2) explains,

Whereas math teachers have traditionally focused on teaching content, more challenging standards are encouraging educators to expand their instruction to promote students' mathematical skills, most of which depend heavily on learning to communicate effectively about math.

Additionally, because content standards are reviewed and revised, frequent shifts are made in grade-level content over time. In contrast, regardless of the content being taught, the practices/processes of mathematical thinking remain constant. For this reason, *Let's Talk Math* focuses specifically on *how* students approach tasks.

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Revisiting the Grade 3 team meeting, the teachers shared that several students in their classes did not know how to get started on the Bicycle-Tricycle task. One teacher noticed that many of her students waited for her to help them get started, while another teacher discussed that several of her students simply took numbers and started calculating, yet their work did not relate to the context in the task. Overall, students struggled to make sense of this complex task and in an effort to do something, took the numbers and applied mathematical operations without persevering to make sense of the quantities in the task or their answers. Students need to understand that problem-solving is not answer-getting.

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## Supporting All Learners

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In order to develop problem solvers, students must spend time practicing. Sometimes teachers skip giving word problems to students who are not native English speakers, who may not find a task relevant, or who often struggle mathematically. This is problematic in that it does not build the skills needed to become efficient and effective problem solvers. Avoidance does not support all learners. Other times, teachers lessen the language in a task to reduce the cognitive load, but that inadvertently takes away the opportunity for students to be exposed to rich language. *Let's Talk Math* offers supports for all learners in the Teacher's Guide so that teachers can feel confident about offering challenging grade-level tasks to all students.

### Language Support

Given that mathematics is a language, it is critical to provide students with proper vocabulary and language development opportunities. This will help students better understand the tasks and apply the mathematics they know. In particular, emerging bilingual students need support in acquiring the language of mathematics while simultaneously acquiring English. However, there are often academic and mathematical terms, or words with multiple meanings, which could prevent any student from accessing the mathematics.

While vocabulary instruction is an essential part of making mathematics content comprehensible for students, language support means more than just vocabulary instruction. Especially helpful for emerging bilingual students is showing images and videos and using gestures, which are essential ways to make sure students understand a language-dense text or problem. Additionally, providing opportunities for guided interaction enables students to develop listening, speaking, reading, and writing skills through structured, academic conversations with their peers. Structured discourse is the heart of *Let's Talk Math*. Giving students opportunities to interact with one another in collaborative groups (with protocols structuring the possible language

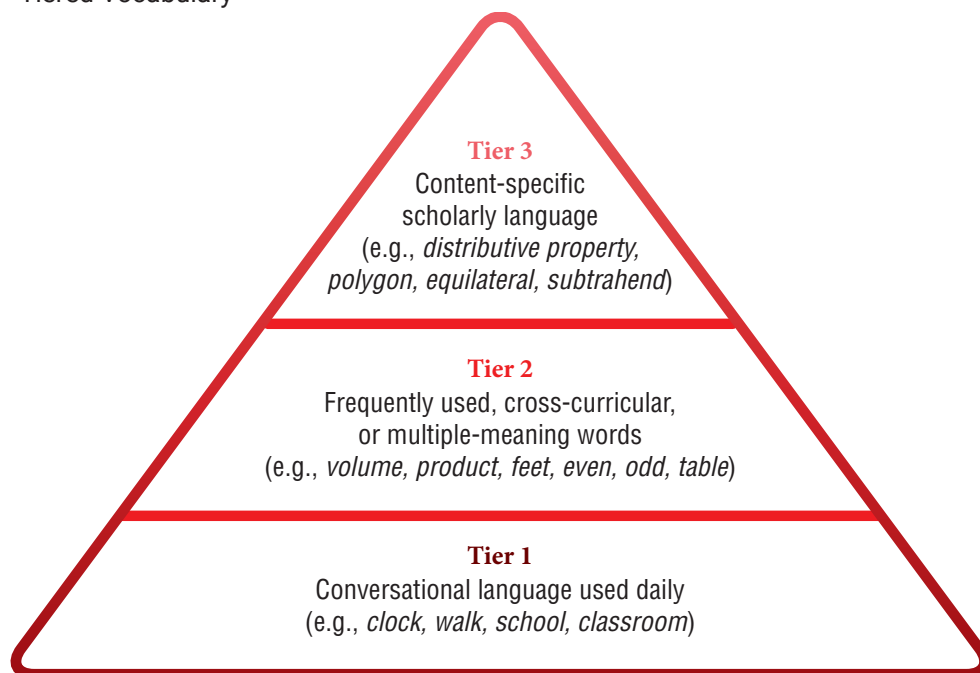
they could use) is an important way to encourage language production. A quality mathematics classroom for all learners, especially emerging bilingual students, is one where student talk is prioritized and heavily scaffolded to ensure comprehension and to optimize output. According to research, students in language-rich environments who solve real-world math problems and use multiple modes of communication develop better English language skills than students in classrooms that did not provide opportunities to experiment with language (Chval and Chavez 2012). *Let's Talk Math* guides teachers in using higher-level questioning skills which can benefit bilingual students.

The WIDA Consortium is a group of states whose mission is to “provide timely, meaningful, and actionable research that promotes educational equity and academic achievement for linguistically and culturally diverse students” (2014). One important contribution of the WIDA Consortium is the English Language Proficiency standards. These standards are universally recognized for their ability to measure academic language development. These standards remind educators that all emerging bilingual students enter mathematics classrooms with their own experiences and knowledge bases, and educators need to build on their knowledge bases to help students successfully collaborate, communicate, and create meaning in mathematics.

### Tiered Vocabulary Instruction

*Let's Talk Math* offers teachers guided support with language development to prevent barriers to accessing the mathematics. Figure 3 shows three tiers of vocabulary. Also included, where needed, are grammatical structure supports that highlight phrases that students will need to produce orally during discussion and/or be able to understand to accurately interpret a math problem.

Figure 3—Tiered Vocabulary



- **Tier 3 words are subject-specific and academic in nature.** These words are the content vocabulary and could be posted on a math word wall for reference throughout a unit. These words are often only used in academic settings (e.g., *exoskeleton* is scientific; *tessellation* is mathematical). Defining these words is essential for providing comprehensible input, in other words, presenting the mathematics content in a way that students can understand.
- **Tier 2 words are arguably the most critical to pre-teach as they are high-utility words that students will be able to use across multiple subject areas (e.g., *analysis, compare*).** Focusing instruction on these words will provide students with invaluable academic language. Tier 2 words also often have multiple meanings and can be confusing for students, especially emerging bilingual students. In addition, these words can help students in their literacy acquisition (e.g., transition words).
- **Tier 1 words are used daily and are generally familiar to most students.** These words tend to be learned through conversation and daily language. They may or may not require explicit instruction but can often be explained with quick visuals, gestures, or demonstrations.

Included for each task are suggested Tier 1, 2, and 3 words, where appropriate. Teachers should intentionally select appropriate tiered words to discuss with students. Selecting vocabulary words for instruction depends on the needs of individual students. For example, some emerging bilingual students at earlier stages of language acquisition will need visual supports and definitions for Tier 1 words to comprehend the content. Other emerging bilingual students, who are further along in their language development process, may be familiar with most Tier 1 words. Tier 2 and Tier 3 words will be essential for most emerging bilingual students, but the teacher may choose not to provide extensive instruction on all the words identified. The teacher may choose to do a quick verbal or visual formative assessment at the beginning of the mathematics lesson to see which students may be unfamiliar with certain words.

There are many ways to incorporate vocabulary development into mathematics instruction. For Tier 3 words, the teacher can post the words and corresponding visuals on a math content word wall during a unit. Students should learn how to use the word wall as an anchor and refer to it when needed. All identified Tier 3 words are provided as word cards in the Digital Resources of *Let's Talk Math*.

## Grammatical Structure

Supporting grammatical structure is equally as important as emphasizing tiered vocabulary. *Let's Talk Math* provides teachers opportunities to help students increase their linguistic complexity output. For example, in a unit of study about fractions, students may be asked to compare fractions by noticing differences. A teacher

may provide the following sentence frame for students to use to help structure their comparisons: \_\_\_\_\_, *but* \_\_\_\_\_. For instance, students may use the sentence stem to say, “The number on top in the first fraction is 1, *but* the number on the top in the other fraction is 2.” After practice, an emerging bilingual student may use this sentence structure with relative ease. A teacher may then provide students with alternative sentence constructions that convey the same meaning, but elevate the language, such as \_\_\_\_\_, *whereas* \_\_\_\_\_. Notice how now, the student would say, “The number on top in the first fraction is 1, *whereas* the number on the top in the other fraction is 2.” Other challenging grammatical structures highlighted within *Let’s Talk Math* are:

**Present tense verbs**—These can often be found in general rules or statements (e.g., A triangle *has* three sides.).

**Modal verbs**—These can often convey the steps of a math application (e.g., You *have to* multiply the length and the width to find the area of a rectangle.).

**Conditional tenses**—These can often showcase a cause and effect relationship (e.g., If you *multiply* any number by zero, then the product *will* always be zero.).

Emerging bilingual students need opportunities to rehearse increasingly complex language structures. Mathematics learning activities that spark collaborative discussion are an essential outlet for such rehearsal.

## Culturally and Linguistically Responsive Pedagogy

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All students should feel successful when learning mathematics. *Let’s Talk Math* is built upon this premise. Mathematics educators must understand who the learners in their classrooms are and value their cultures, experiences, and native languages to ensure that instruction does not limit opportunities for students to succeed.

In part, being linguistically responsive means that educators are familiar with their students’ backgrounds, how students’ home languages play a role in their education, and how to properly scaffold or adjust the language of a task to make it accessible. This entails understanding the language demands of the tasks. As previously discussed, *Let’s Talk Math* highlights tiered vocabulary and grammatical structures for each task that might impede a student’s ability to access the mathematics. It is critical that teachers have a clear





understanding of a student’s level of standard English proficiency and familiarity with their language background to best address their needs. The bottom line is that a lack of fluency in standard English should not prevent a student who is developmentally ready to learn the content from doing so.

Being culturally responsive means that educators include students’ cultural experiences in all facets of learning and make the learning opportunities relevant and effective for them (Gay, 2010; Ladson-Billings, 1994). Mathematics teachers have an opportunity to help students see mathematics as a tool. Because of this, educators must be aware and responsive to their personal biases (implicit and/or explicit) and the ways in which they serve students. Ideally, mathematics should serve as a window through which students can evaluate and understand their environments, while also serving as a mirror to reflect on their understandings, their strengths, and areas in which they can improve (Gutierrez, 2012). Culturally responsive teaching is about students seeing themselves as part of the learning (Hammond, 2015). To help students see themselves in this resource, educators must choose tasks intentionally. The tasks in *Let’s Talk Math* were intentionally designed to meet a broad spectrum of experiences, represent a variety of cultures and environmental contexts, and challenge stereotypical situations. Language-rich instruction with a focus on high-quality math instruction will support all students, but especially those who are culturally diverse (Araujo, Roberts, Willey, and Zahner, 2018).

## Differentiation

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Another aspect of supporting all learners is differentiating instruction to meet learners’ needs. While teachers should allow students to productively struggle and persevere through challenges, at times, students’ thinking may need to be guided by providing additional support. Suggested scaffolds, extensions, and possible misconceptions accompany each task in *Let’s Talk Math*. Each task offers suggestions for working with students who demonstrate a need for additional support and extension opportunities for students who are ready for additional challenges. This resource supports all levels of learners.

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Returning to the Bicycle-Tricycle task again, just as Teacher 3 adjusted the numbers in the task and lowered the cognitive demand, adjusting the language to eliminate challenging words, or changing the context of the task if not relevant to students can prevent students from diversifying their experiences. Instead, *Let’s Talk Math* offers guidance for how to support all learners to be successful in understanding a task. In the case of the Bicycle-Tricycle task, multiple meaning words such as “shop” would be highlighted in the Teacher’s Guide as a tier 2 word, noting that teachers should pre-teach this word before students engage in the problem-solving process to increase their chances for success. In addition, since bicycles and tricycles may not be a familiar context for all students, images of the items would be presented on the student task card to contextualize the situation. Ultimately, supporting all learners is about providing them opportunities to learn and be successful.

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### Guiding Principle #2:

Communicating mathematically encompasses speaking, listening, reading, and writing effectively.

Communicating mathematically means that students are able to speak, listen, read, and write in order to share ideas, clarify understanding, and build meaning for ideas (National Council of Teachers of Mathematics 2000). In fact, Thompson and colleagues recommend that educators “consider every student a *mathematics language learner* regardless of his or her level of English

language proficiency” (2008, 11). This is important, as educators often find that their students, when presented with a task, **number grab** to solve problems, and/or forget to pay attention to the context of the problem. To remedy that, students are often taught to look for key words, which can be misleading if students are not making sense of the problem. *Let’s Talk Math* helps students slow down, comprehend the situation, and communicate about the math. To support classroom discourse, as well as reading and writing, *Let’s Talk Math* uses routines with structured protocols to support students in learning to speak, listen, read, and write mathematically. According to Kelemanik, Lucenta, and Creighton, “Instructional routines are meant to be repeated, and this repetition makes them very effective vehicles for developing mathematical practices” (2016, 4). Through *Let’s Talk Math*, students engage in repeated routines that they begin to internalize and make use of independently.

**number grab:** pulling numbers out of a word problem, often ignoring the text, and performing a mathematical operation

## About the Routines

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*Let’s Talk Math* is centered on three instructional routines often referred to as “the 3 steps for problem-solving success.” These routines create structured discourse opportunities in mathematics classrooms and help students optimize their output, both in problem-solving and in the use of language. Cameron, Gallahue, and Iacoviello (2020) also emphasize the importance of mathematical routines, specifically routines that have predictable patterns and are meant for students to remember. Problem-solving, much like storytelling, has three parts: beginning, middle, and end. The routines in *Let’s Talk Math* help tell that story. The first routine is called Understand and Plan. The second routine is Share and Discuss. The final routine is Reflect and Write. Each of the six mathematical practices/processes in *Let’s Talk Math* have unique routines for each type of interaction described below. *Let’s Talk Math* provides two versions of the 3 Steps for Problem-Solving Success. As K–1 students may be emerging readers, the teacher guides and supports students in using the routine.

## **Understand and Plan Routine**

Typically, students are presented with a mathematical task without having an opportunity to collaborate to understand the task itself *before* solving. The Understand and Plan routine provides students the time, while working with partners, to read the task aloud, rephrase the task at least once, identify important information, and plan how they might attempt to solve the task by making sense of the task together. This is all done collaboratively. This initial routine eliminates general barriers to the mathematics, such as language, and offers support and reassurance for students as they know that they are not alone as they face a challenging task. The Understand and Plan routine also affords students a chance to make their own meaning and develop familiarity with academic or high-utility, multiple-meaning vocabulary words that are not used in everyday conversations. Once students have had a chance to make sense of the task together, they solve it independently.

## **Share and Discuss Routine**

After independently solving the task, students come back together to share their work. Once again, they are asked to share what they heard their partners say. Having students repeat what they heard their partners say teaches them that listening carefully to others enhances their understanding of what has been said. Rephrasing enables the first speaker to feel confident that their message was clear or provides opportunities for clarification as needed. Knowing that their partner has understood their statements contributes to the speaker's self-confidence and pride as a contributor to their shared problem-solving process. It also allows for every student to share and contribute equally. The goal is to listen closely to their partners' problem-solving processes so they can then discuss similarities and differences. This helps students understand multiple problem-solving strategies and draw connections between their mathematical ideas.

## **Reflect and Write Routine**

After students have completed the Share and Discuss routine, they engage in the Reflect and Write routine. This framework helps students focus their attention on a particular mathematical practice/process. It also gives them an opportunity to review their work to identify how they interacted with the specific mathematical practice/process. The Reflect and Write routine provides students opportunities to deepen their mathematical understandings. Sometimes, students might solve a task and not notice they engaged in a mathematical behavior. The goal of the Reflect and Write routine is for students to slow down and recognize the practice/process that might have been overlooked while solving the task. Students should be able to repeat those habits when problem-solving at other times. After reflecting on a particular mathematical practice/process they used, students are then asked to write their individual reflections.

## Example of Routines

One of the mathematical practices/processes included in *Let's Talk Math* is *Think Using Quantities*. The following example of the routines is specific to that mathematical practice. Each practice/process has unique protocols and therefore other practices/processes vary from this one.

### Understand and Plan Routine for *Think Using Quantities*

#### K–1 Grade Level Version

**Teacher reads the task aloud.**

**Student 1:** “The story is about \_\_\_\_\_.”

**Student 2:** “One quantity I see is \_\_\_\_\_.”

**Student 1:** “I also see \_\_\_\_\_.”

**Student 2:** “The task asks us to \_\_\_\_\_.”

**Both:** Solve the task independently.

#### 2–5 Grade Level Version

**Student 1:** Read the task aloud.

**Student 2:** “What I heard you say is \_\_\_\_\_.”

**Student 1:** “The question we need to answer is \_\_\_\_\_.”

**Student 2:** “The important information is \_\_\_\_\_.”

**Student 1:** “For this task, we need to focus on using quantities.”

**Student 2:** “To think using quantities in this task, we can \_\_\_\_\_.”

**Both:** Solve the task independently.

### Share and Discuss Routine for *Think Using Quantities*

#### K–1 Grade Level Version

**Both:** Come back together to share your work.

**Student 2:** “First, I \_\_\_\_\_. Then, I \_\_\_\_\_.”

**Student 1:** “What I heard you say is \_\_\_\_\_.”

**Student 1:** “First, I \_\_\_\_\_. Then, I \_\_\_\_\_.”

**Student 2:** “What I heard you say is \_\_\_\_\_.”

#### 2–5 Grade Level Version

**Both:** Come back together to share your work.

**Student 2:** “First, I \_\_\_\_\_. Then, I \_\_\_\_\_.”

**Student 1:** “What I heard you say is \_\_\_\_\_.”

**Student 1:** “First, I \_\_\_\_\_. Then, I \_\_\_\_\_.”

**Student 2:** “What I heard you say is \_\_\_\_\_.”

**Both discuss:** “How are our strategies similar or different?”

## Reflect and Write Routine for *Think Using Quantities*

### K–1 Grade Level Version

This part of the protocol is teacher-led for kindergartners and first graders as many students are beginning writers.

### 2–5 Grade Level Version

**Student 1:** “Which quantities were important in this task?”

**Student 2:** *Respond.*

**Student 2:** “What did we notice about the important quantities?”

**Student 1:** *Respond.*

**Both reflect:** “How did we use the quantities to help us solve the task?”

**Both write:** “We used the quantities to help us solve by \_\_\_\_\_.”

## Speaking and Listening

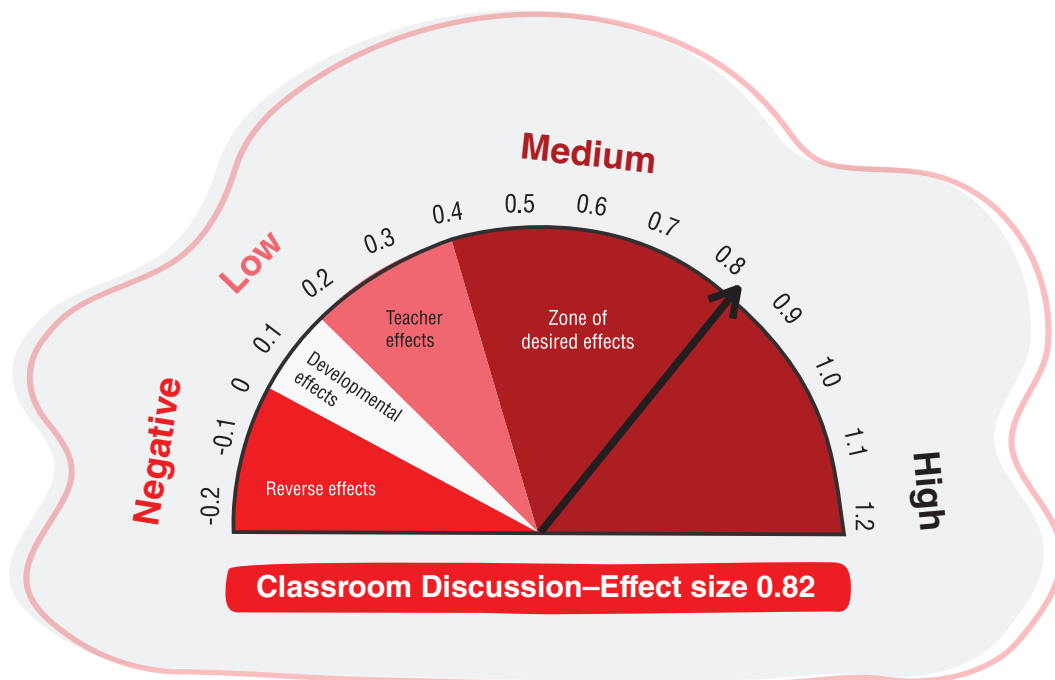
Classrooms that support twenty-first century learners look like collaborative spaces, not assembly lines. Innovation and creative thinking are best produced when students collaborate, rather than work in isolation. *Let’s Talk Math* provides opportunities for students to work together to make sense of mathematical tasks while simultaneously thinking critically and creatively about their problem-solving processes.

Research has shown that discourse is fundamental to mathematics learning. In 2014, the National Council of Teachers of Mathematics (NCTM) published *Principles to Actions*, which outlines evidence-based beliefs about effective mathematics teaching practices. According to NCTM, effectively teaching mathematics requires asking purposeful questions that enhance students’ reasoning skills while also advancing their abilities to make sense of mathematical ideas and relationships. Additionally, Carpenter, Franke, and Levi state, “Students who learn to articulate and justify their own mathematical ideas, reason through their own and others’ mathematical explanations, and provide a rationale for their answers develop a deep understanding that is critical to their future success in mathematics and related fields” (2003, 6).



Similarly, John Hattie synthesized meta-analyses to understand those teaching practices that have the highest impact on student learning (Hattie, Fisher, and Frey 2017). Hattie developed a ranking system of effect sizes based on the research to determine the actions that have the greatest impact. As seen in Figure 4, an effect size of 0.1 has little impact on student learning and simply refers to developmental growth, while an effect size of 0.4 indicates student learning occurring at an appropriate rate: one year’s growth in one year’s time. Hattie then focused on those teaching practices that promote students’ learning at a greater rate than 0.4 effect size. The findings? Classroom discussion has an effect size of 0.82—double the rate of student learning!

Figure 4—Teacher Actions That Affect Student Learning (Hattie 2009)



*Let’s Talk Math* builds on Hattie’s research. It provides students with structures that allow them to think about mathematical tasks in specific ways. Students work through their thinking with partners using three specific routines: Understand and Plan; Share and Discuss; and Reflect and Write. Together, they make sense of each task and then work independently to solve the problem. Then, students compare and discuss their thinking by using specific protocols that help frame their thinking. Finally, students engage in the Reflect and Write routine. Students reflect on the specific practice used in the task and then they write their own summary independently. *Let’s Talk Math* models the strategies presented by these researchers by encouraging students to engage in practical discourse with one another.



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During the Grade 3 team meeting, Teacher 2 brought video footage of her students working through the Share and Discuss routine for the Bicycle-Tricycle task, a task focused on the mathematical practice of *Thinking Using Quantities*. The following is an excerpt from the students' conversation as they use the *Let's Talk Math* protocols to share their work.

**Student 2:** First, I drew a box with circles for the bicycles and tricycles. Then, I counted the circles as I went. I think that there could be 6 bicycles, but that would mean that there are no bicycles. I also think that there could be 3 bicycles and 4 tricycles because that makes 18 wheels.

**Student 1:** What I heard you say is that you drew a picture using a box with circles. You counted a lot. You got two answers: you saw that there could be 3 bicycles and 4 tricycles because that makes 18 wheels, or there could be 6 tricycles and no bicycles.

**Student 1:** First, I made a chart. I realized that some groups of bicycles and tricycles didn't work at all—like we can't have 1 bicycle because that left 16 wheels for the tricycles and tricycles have 3 wheels, so there would be 1 leftover wheel. I found the same answers you did and I also saw that 6 bicycles and 2 tricycles could work.

**Student 2:** What I heard you say is that you made a chart and you found 2 ways that made 18 wheels that were different from my ways.

Students then compared how their strategies were similar and/or how they were different. Engaging in this routine using protocols to structure the conversations enable students to communicate effectively about their work.

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This idea that a classroom of elementary mathematics learners should be led by and centered on their justifications, reasonings, and discussions is the backbone of mathematical discourse. Classrooms today must prepare students for the twenty-first-century workforce, which is a collaborative and innovative space where people must be able to communicate their ideas clearly and concisely, listen carefully to others' thinking and strategies, determine how to compromise and move forward collectively, and continue discussions so that all perspectives are valued.



For students to be prepared for a collaborative work environment, they must learn how to engage appropriately in discussions. This should start as early as kindergarten. For students to engage successfully in discourse, we need to explicitly teach students how to have conversations where they share ideas and listen to others. *Let's Talk Math* provides structured routines that strategically focus on developing and promoting students' active listening skills in mathematics. To cultivate such a classroom climate, teachers must provide students opportunities and time to talk with one another. As Klaus-Quinlan and Nathenson-Mejía state, "Language development is an active, not passive process" (2010, 16). Establishing a mathematics classroom focused on collaborative problem-solving enables students to develop the language skills they need to communicate effectively both within and beyond mathematics. Such a mathematics classroom places the responsibility for learning squarely on the shoulders of the students.

## Reading and Writing

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One of the most critical features of *Let's Talk Math* is the writing-in-mathematics component. According to Amy Benjamin, "Writing causes learning. By writing, we create, transform, mobilize, integrate, and secure what might otherwise be fragile knowledge" (2013, xxii). Further, Dacey (2018) adds that writing in mathematics helps students create and show knowledge, draw connections between conceptual understanding and procedural fluency, and communicate mathematically. As such, each and every task in *Let's Talk Math* requires students to reframe their thoughts into written responses, further integrating mathematics and literacy. The Common Core standards, as well as national and state-level standards, provide model frameworks for integrating writing and mathematics so that teachers can link key innovative processes and strategies to strengthen students' skills in both subjects (Brozo and Crain 2018). *Let's Talk Math* supports students in improving both mathematics and writing skills.



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Coming back to the Grade 3 team meeting, Teacher 2 continued to showcase her students' engagement using the routines and protocols. After students engaged in the Share and Discuss routine, where they shared their approaches and solutions to the task, students then engaged in the Reflect and Write routine, where they summarized their work and application of the mathematical practices/processes.

**Student 1:** What quantities were important in this task?

**Student 2:** There were 18 wheels and we had to keep track of the number of wheels on the bicycles and tricycles.

**Student 2:** What did we notice about the important quantities?

**Student 1:** We noticed that some groups of bicycles and tricycles did not work to make 18 wheels.

Both students then reflected together on how they used quantities to help them solve the task. They then wrote their own individual reflections:

**Student 1:** *We used quantities to help us solve the task by making a chart. I noticed that some quantities of bicycles and tricycles didn't work and that there could be all bicycles or all tricycles. I also knew that 3 bicycles and 4 tricycles and 6 bicycles and 2 tricycles both were combinations that could work.*

**Student 2:** *We used quantities to help us solve the task by drawing pictures of the bicycles and tricycles and their wheels. That took me a long time. I saw that 3 bicycles and 4 tricycles gave me 18 wheels.*

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## Standards Overview

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The Every Student Succeeds Act (ESSA) mandates that all states adopt challenging academic standards that help students meet the goal of college and career readiness. While many states adopted academic standards prior to ESSA, the act continues to hold states accountable for detailed and comprehensive standards. Standards are designed to focus instruction and guide adoption of curricula. They define the knowledge, skills, and content students should acquire at each level. Standards are also used to develop standardized tests to evaluate students' academic progress. State standards are used in the development of our resources, so educators can be assured they meet state academic requirements. As required by ESSA, all students must have access to high-quality math instruction to increase their college and career readiness skills.

*Let's Talk Math* is correlated to the academic standards of all 50 states, the District of Columbia, the Department of Defense Dependent Schools, and the Canadian provinces. A correlation is also provided for key professional educational organizations.

## Conclusion

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Mathematical problem-solving is a challenging aspect of instruction for many mathematics educators. Research has found that classroom discourse plays a significant role in student achievement (Hattie 2009). *Let's Talk Math* builds on this research, and more, to provide students opportunities to engage in structured mathematical discourse as a means to enhance problem-solving skills. Through structured routines that strategically focus on developing mathematical habits of mind, students learn how to make sense of a task and persevere when challenged. Through this supplementary research-based resource, teachers can feel confident that their students are receiving high-quality mathematics tasks and experiences that they will be able to apply to new circumstances. In fact, during pilot testing, teachers noticed that students who learned the routines often referred to using the mathematical practices in other contexts, as well as using the structured protocols for problem-solving tasks outside of *Let's Talk Math*. By doing so, students exhibited their ability to transfer their knowledge to new topics and situations. Teachers were often impressed with students who surprised them by what they were able to accomplish. While teachers shared the impact *Let's Talk Math* could have on students' mathematics learning, students also offered feedback. One student said, "I like doing the tasks because I get to see how my brain thinks." A fourth grade student shared, "It was really nice that I could talk with a partner and have conversations with them. Also, I liked the structured conversations ... usually I completely can't say... my mind goes blank, and I have trouble thinking of things to say. When they give us those sentences already started with blanks I think it's a bit easier for me to talk with my partner because then I just need to put my work into those sentences."

*Let's Talk Math* engages students as mathematical thinkers and enhances any core mathematics curriculum.

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